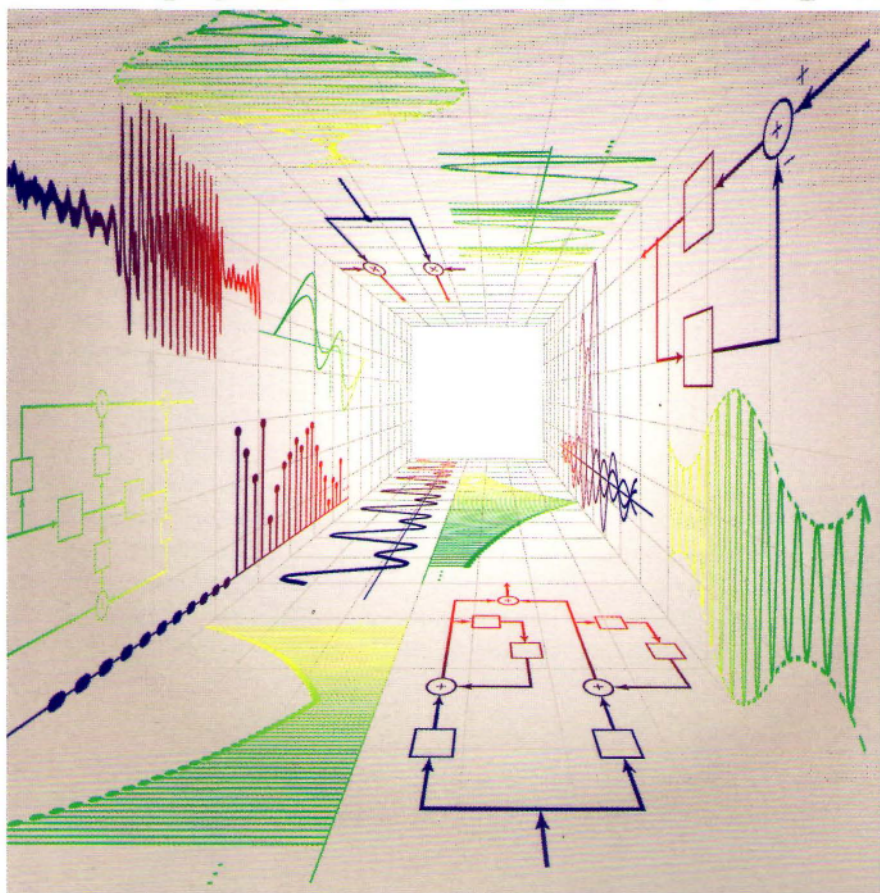


INTERNATIONAL EDITION

SIGNALS & SYSTEMS

SECOND EDITION



ALAN V. OPPENHEIM
ALAN S. WILLSKY
WITH S. HAMID NAWAB

PRENTICE HALL SIGNAL PROCESSING SERIES
ALAN V. OPPENHEIM, SERIES EDITOR



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SIGNALS & SYSTEMS

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